

A Statistical Investigation of the Contamination of Binary Gravitational Lens Candidates by Cataclysmic Variables

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Abstract

A common remark at conferences and meetings has been: “One can fit *any* light curve with binary lens models because of the large number of parameters!” It is especially tempting to claim that some of the possible binary events in the Galactic microlensing searches could be caused by cataclysmic variables since both phenomena exhibits asymmetric light curves. This paper presents a statistical analysis comparing the light curves of cataclysmic variable stars to the light curves generated from binary lens models. It is shown that the light curves of high amplitude cataclysmic variables are one example of light curves that can *not* be explained with binary lens models. It is, therefore, not only unlikely, but also impossible that cataclysmic variables can contaminate the binary microlens candidates.

Subject headings: Gravitational lenses — novae — cataclysmic variables

1 Introduction

Applications of gravitational lens theory was recently expanded when first Paczyński (1986) predicted the possibility of detecting dark matter in the Galactic halo by searching for microlensing effects on millions of stars in the LMC, and, by now, the four collaborations: The MACHO group (for the latest results see the world wide web at: wwwmcho.mcmaster.ca/ or wwwmcho.anu.edu.au/), the EROS group (Aubourg et al. 1993), the OGLE group (see www.astro.princeton.edu/~ogle/ or www.astrouw.edu.pl), and the DUO group (Alard 1995) have

detected more than 100 microlensing candidates. To distinguish between variable stars and microlensing a number of criteria was used, one of which was that the light curve be symmetric, i.e. only microlensing by a single object was considered.

However, as the observations have shown, binary lenses do indeed exist; notable examples being the DUO 2 (Alard et al. 1995), OGLE 7 (Udalski et al. 1994), and the LMC binary event (Bennett et al. 1996) candidates. A priori, binary lensing is expected in about 10% of lensing events (Mao & Paczyński 1991), and Mao & Di Stefano (1995) developed a method that determines the lensing parameters from a given light curve. This method was successfully applied to DUO 2 (Alard et al. 1995) and OGLE 7 (Udalski et al. 1994) showing that their light curves are fit well by binary lens models; thereby, raising the question of how many of the objects previously believed to be variable stars could, in fact, be microlensing candidates. In addition, Dominik & Hirshfeld (1995) showed that the light curve of the MACHO LMC # 1 candidate could be explained better by a binary lens model than a single lens model. A recent review of the Galactic microlensing searches can be found in Paczyński (1996).

In an investigation similar to this one, Della Valle & Livio (1995) found that the distribution of the amplitudes relative to the durations of ordinary dwarf nova eruptions is significantly different from those exhibited by the main body of the microlensing candidates from the MACHO and OGLE collaborations. They concentrated on single lens models which limits the investigation to cataclysmic variables with symmetric light curves. They concluded that cataclysmic variables with symmetric light curves are not a significant source of contamination, but speculated that the most serious source of potential contamination may be due to old novae that exhibit dwarf nova outbursts. By adopting the current estimates for the space density of such old novae, they found that a significant fraction of the microlensing candidates could be cataclysmic variables.

In this paper, the possibility of confusing microlens candidates and cataclysmic variables is investigated further, and, since Della Valle & Livio (1995) estimated that only 20-40% of cataclysmic variables have symmetric light curves, only binary lens models, which can produce asymmetric light curves, are considered. Because of the well sampled light curve and its frequency-amplitude relation, SS Cygni is used as a typical cataclysmic variable, and the features of SS Cygni is compared to light curves caused by binary microlenses.

2 Basic Theory

Since the theory of gravitational microlensing has been presented in detail by numerous authors (e.g. Schneider et al. 1992 or Paczyński 1996), only the most basic ideas and notation are included in this section. Assume that a point source is located at an angular diameter distance, D_s , and a binary lens at distance D_l from the observer. Let a light ray from the source pass at a distance R_i from the i 'th lens with mass M_i . Define the dimensionless mass of the i 'th lens as:

$$m_i = \frac{(D_s - D_d)}{D_s D_d} \frac{4GM_i}{c^2} \quad (1)$$

and define $m_1 + m_2 = 1$ which makes all angles measured in units of the Einstein ring radius, R_E , which is given by:

$$R_E = \left(\frac{4GD_l D_{ls}}{D_s c^2} \right)^{\frac{1}{2}} \quad (2)$$

for a lens with unit total mass. D_{ls} is the angular diameter distance between lens and source. R_E is a few AU for lensing in the LMC or the Galactic halo. The lens equations for a binary lens then becomes:

$$x_s = x - \frac{m_1(x - x_1)}{r_1^2} - \frac{m_2(x - x_2)}{r_2^2} \quad (3)$$

$$y_s = y - \frac{m_1(y - y_1)}{r_1^2} - \frac{m_2(y - y_2)}{r_2^2} \quad (4)$$

where (x_s, y_s) , (x, y) , and (x_i, y_i) are the angles of the source, the image of the source, and $i'th$ lens, respectively, as seen by the observer, and r_i is the impact parameter in the observer plane. The theoretical light curves can then be computed from the lens equations (3) and (4) using the method described in Witt (1990) which simplifies the lens equations into a fifth-order polynomial, which in turn can be solved numerically by Laguerre's method (see Mao & Di Stefano 1995).

3 Numerical Method

As can be seen from the previous section, the light curve of a background point source created by a binary gravitational lens can be characterized by six variables: The (four) positions of the two lenses in the observer plane ((x_1, y_1) and (x_2, y_2) all measured in units of R_E), the mass ratio of the lenses ($q = m_1/m_2$), and the time parameter $T_E = R_E/V$ measured in days. It is not necessary to solve for the source position, since changing (x_s, y_s) is equivalent to move the lenses. Therefore, define y_s as 0.0 and let x_s vary between $[-2.5, 2.5]$. In addition, it is necessary to know the baseline, i.e. the magnitude of the unlensed source, and which fraction of the observed light is caused by nearby unlensed sources. These two parameters can be estimated by other means than including them as free parameters, and they are, therefore, treated as input parameters. It is not possible to analytically solve for the six parameters for a given light curve, and it is, therefore, necessary to fit the observed and modeled light curves numerically. This can be done by minimizing χ^2 given by

$$\chi^2 = \sum_{i=1}^{N_{obs}} \left(\frac{\mu_i - \mu_{i,obs}}{\sigma_i} \right)^2, \quad (5)$$

where N_{obs} is the number of observations, μ_i and $\mu_{i,obs}$ are the theoretical and observed *magnifications*, and σ_i is the measurement error. For large numbers of observations, χ^2 is a normal distribution with mean $N_{obs} - N_{parameters}$ and standard deviation $(2(N_{obs} - N_{parameters}))^{\frac{1}{2}}$ (see e.g. Press et al., 1992). Due to the nature of microlensing light curves, which can have several sharp peaks appearing and disappearing with only small changes in the parameters, χ^2 has many local minima. Using a simulated light curve (with $\sigma_i = 1$ for all i), fig. 1 shows how χ^2 changes as a function of one parameter, the coordinate of one of the lenses. As can be seen from the

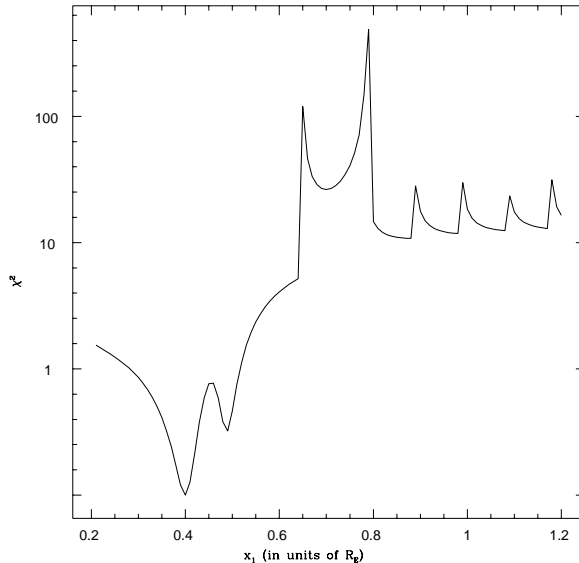


Figure 1: χ^2 as a function of the x coordinate of one of the two lenses. Notice the many local minima at $x_1 = 0.51, 0.67, 0.87$ etc. and the global minimum at $x_1 = 0.40$.

figure, the curve shows numerous local minima which makes it impossible to use standard numerical methods for minimizing a function (for instance the Downhill-Simplex method or Simulated Annealing, which is even designed to disregard local minima, but fails in this case, see e.g. Press et al. 1992).

Looking closely at fig. 1 notice that: As the function gets closer to the global minimum, the local minima get smaller. It would seem that a “simulated tunnel effect” could jump from one local minima to another, since most of them are closely spaced. Using these observations, a numerical method was developed that first uses the Downhill-Simplex Method to find a (local) minima, and then simulates a tunnel effect by randomly searching a six dimensional cube for nearby (local) minima that are smaller than the present minima. To overcome features like the double peak at $x_1 = 0.63$ and 0.79 in fig. 1, the cube is gradually expanded as long as no smaller minima is found. When a new (and smaller) minima is found, the whole process repeats itself. The procedure is terminated when the cube reaches a specified maximum size. When the cube is expanded sufficiently slow and far, this method reliably finds the global minimum using simulated light curves. However, it turns out to be time consuming if the initial guess of the six parameters is far from the global minimum. To increase the efficiency of the program, the initial guesses are determined by defining a grid in the 6 dimensional space, finding the (local) minimum for each initial grid point, and then use the parameters of the best minimum as initial guesses (which, depending on how fine the grid is, may well be the global minimum).

As a test of the program, it was applied to the observations of DUO 2 (Alard et al. 1995) in the B band, which has the most complex light curve of the microlensing candidates so far, and, therefore, should be the most difficult to fit. Assume a 30% blend from an unlensed source and adopt a baseline of $m_B = 21.45$ mag from the unlensed part of the light curve. The first attempts

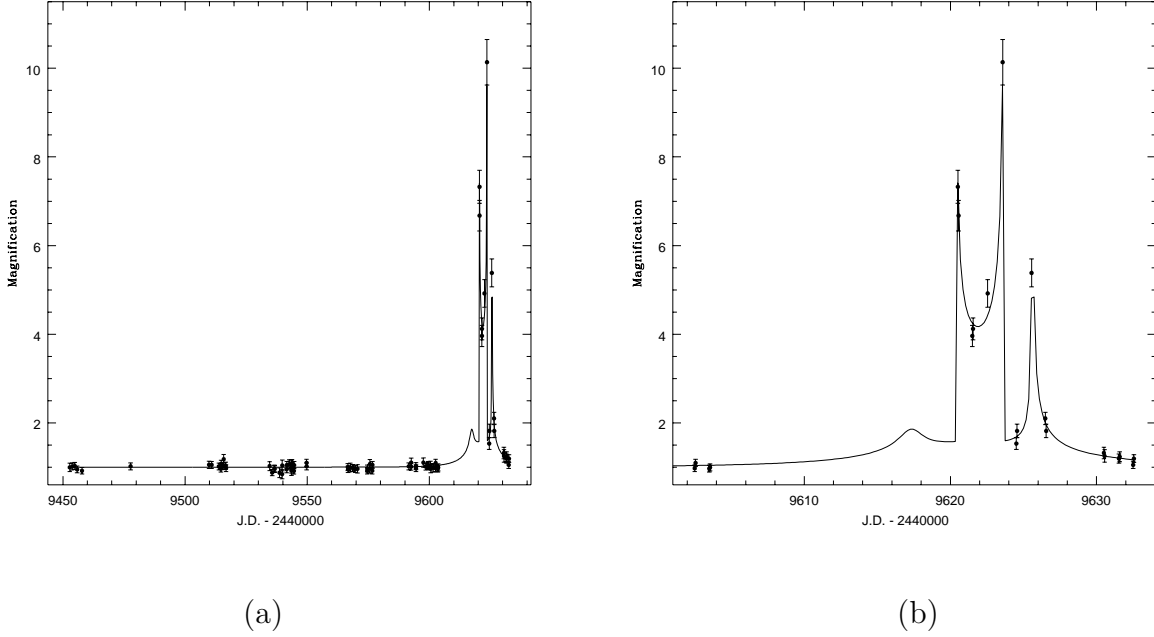


Figure 2: The B band light curve of DUO 2. The solid line represents the best model. (a) All the available data. (b) The lensed part of the light curve.

at fitting the observations showed that the quoted measurements errors were overestimated, and the errors were instead obtained from the scatter in the unlensed part of the curve. In this case, the best fit has a χ^2 of 78 for 80 data points (74 degrees of freedom). Using the same notation as above, the best fit parameters are:

$$\begin{aligned}
 x_1 &= 0.42 & y_1 &= 0.49 \\
 x_2 &= 0.54 & y_2 &= -0.73 \\
 q &= 0.33 & T_E &= 8.85 \text{ days}
 \end{aligned} \tag{6}$$

from which the time of closest approach, T_c , can be calculated: $T_c = 9621.7084$ J.D. - 2440000 or with the notation from Alard et al. (1995): $T_c = 85.06$ J.D. - 2449536.64861. For direct comparison, the parameters used by Alard et al. (1995) are introduced: the distance between the two lenses, a (in units of R_E), the closest approach to the center of mass, b (in units of R_E), and the angle between the binary and the trajectory, θ , this model yields:

$$a = 1.23 \quad b = 0.47 \quad \theta = 92.9^\circ \tag{7}$$

The observations and the best fit model are shown in fig. 2. This result is remarkably close to the solution found by Alard et al. (1995). The best fit parameters only deviate a few percent from each other which is surprising since there's neither no guarantee that the global minimum is found nor is the solution unique. In addition, Alard et al. (1995) fitted both the R and B band data, while only the B band data was considered here and included a finite source size. It should

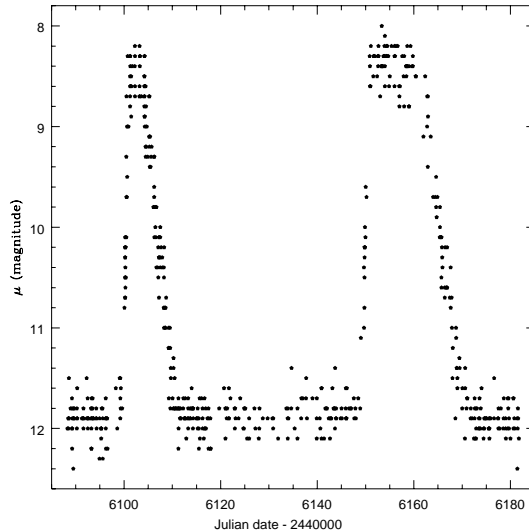


Figure 3: A small part of the light curve of SS Cygni from Mattei et al. (1991).

also be noted that Alard et al. (1995) used a different minimizing method, namely Levenberg-Marquardt's method. The Downhill-Simplex method was used here because it is more robust (but slower). Notice that the method, described in this paper, does not rely on any human decisions once the fitting parameters (maximum size of cube etc.) are determined; the same program can be applied to any light curve without changes or decisions on the best guesses of the parameters. In particular, Alard et al. (1995) had to estimate T_c , and T_E as an intermediate step. By fitting all six parameters simultaneously, this was avoided here.

4 Observations

Assume an unusual (and unlikely) scenario: Let the cataclysmic variable exhibit one event during the observing run, let that run be shorter than normal for microlensing searches, say one month, and assume that subsequent runs happens to be in between bursts. This will give a worst case scenario for contamination of binary lens candidates. Therefore, only cataclysmic variables with an outburst frequency larger than one month are considered in the following. The relation between amplitude and frequency is given by the Kukarkin-Parengo relation:

$$A_{min} - A_{max} = 0.70 [\pm 0.43] + (1.90 [\pm 0.22]) \log(T) \quad (8)$$

where $A_{min} - A_{max}$ is the amplitude of the burst in magnitudes and T is the elapsed time between bursts in days (see e.g. Warner 1995). A one month frequency then corresponds to an amplitude of about 3.5 magnitudes which again corresponds to a magnification of a factor of 25. Notice that, according to eq. (8), a low amplitude variable would exhibit several events during the same observing run, and would, therefore, immediately be classified as a variable star.

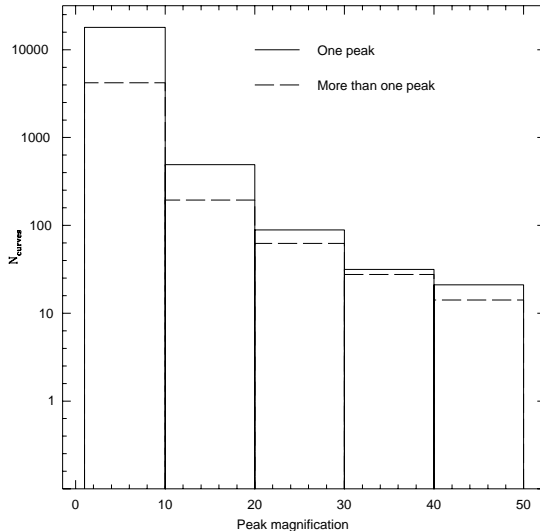


Figure 4: The number of light curves, N_{curves} , as a function of the peak magnification. Both light curves with one peak and two or more peaks were included in this figure.

Based on the above, a good example of a cataclysmic variable is then SS Cygni, a dwarf nova of U Geminorum type, with an average outburst frequency of 50 days and an amplitude corresponding to a magnification of a factor of 30. The American Association of Variable Star Observers combined observations from numerous people (see Mattei et al. 1991, which can be found at www.aavso.org/monographs.html, for details and acknowledgments) thereby creating a well sampled light curve of SS Cygni over a long period of time. A small part of this light curve is shown in fig. 3.

5 Results

As we have seen in the previous sections, only high magnification events (defined here conservatively as more than a factor of 10) are of interest. According to Schneider et al. (1992), the probability that the magnification of a point source is larger than a certain value, μ_o , is proportional to μ^{-2} independent of lens model. Since only single peak light curves have any interest, $\approx 250,000$ light curves and their peak magnification using binary lens models were calculated. The resulting histogram is shown in fig. 4. As can be seen from the figure, of the total 250,000 light curves only $\approx 3\%$ are high magnification events with one peak. The possibility of mistaking cataclysmic variables as binary lens candidates is, therefore, exceedingly small; out of the total of ≈ 100 microlensing candidates, one would expect $\approx 10\%$ to show binary lens signatures (Mao & Paczyński 1991), and only 3% of these would exhibit light curves similar to cataclysmic variables.

However, if the light curve is well sampled, it is actually impossible to confuse the two phenomena. Fig. 5 shows the shape of one event in SS Cygni and a light curve of a high magnification event on the same scale. As can be seen, the two light curves are incompatible with each other,

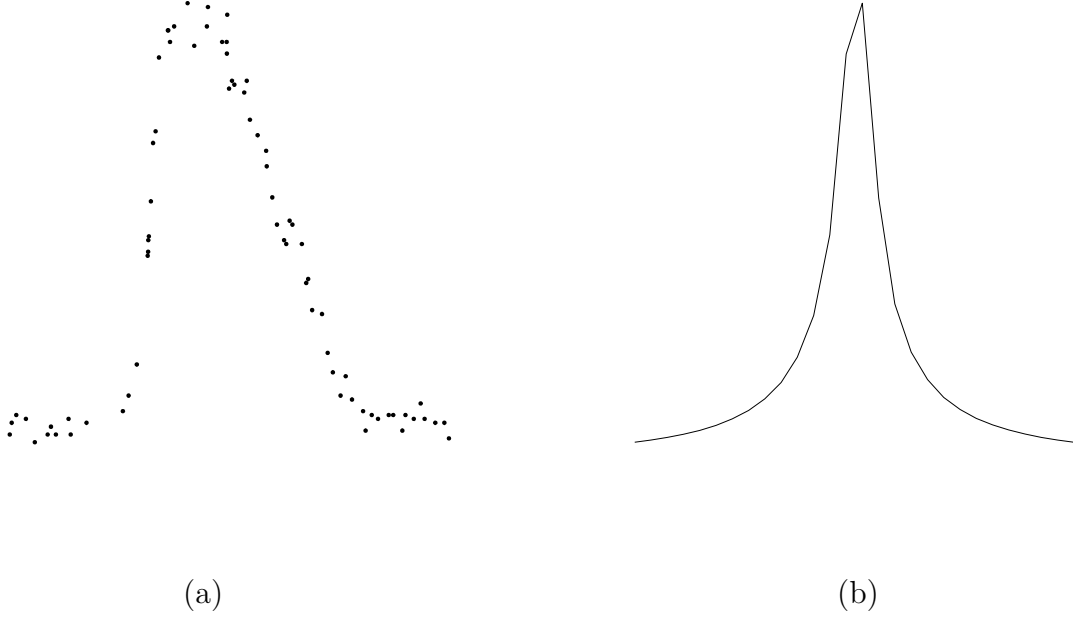


Figure 5: The typical shape of an event for: (a) SS Cygni. (b) Models with one peak and a peak magnification of 10 or more.

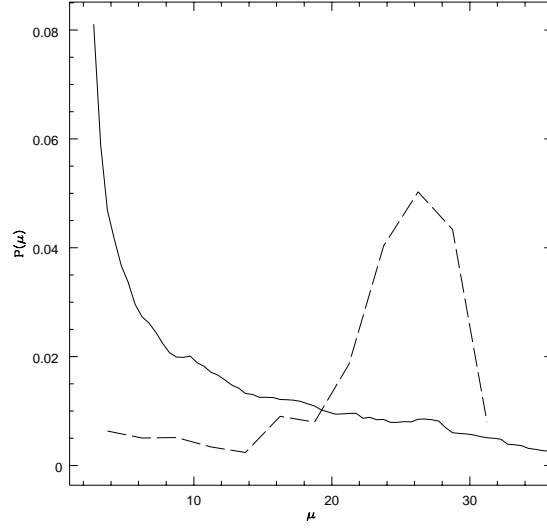


Figure 6: The probability of the magnification being in the interval $[\mu - 1.25, \mu + 1.25]$ for models fitting the one peak, factor of ten or more magnification criteria (solid line) and all the SS Cygni events from Mattei et al. (1991) (broken line).

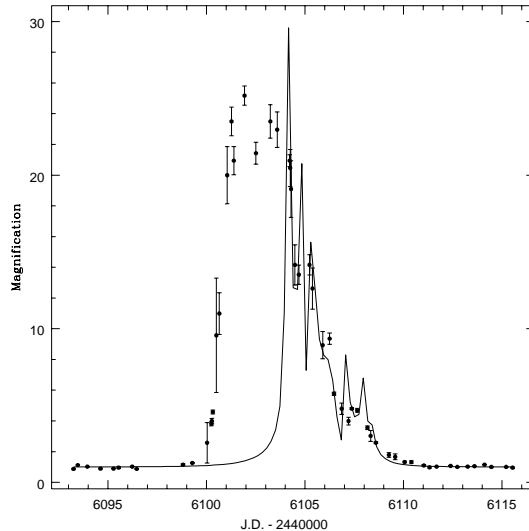


Figure 7: The best fit to an event in SS Cygni.

especially in the wings. To quantify this statement, the probability of a magnification interval $\Delta\mu$, was calculated from all events in Mattei et al. (1991) and all models fitting the more than a factor ten, one peak criteria. To avoid numerical artifacts from the large scatter in the SS Cygni observations, $\Delta\mu$ was set equal to a magnification of a factor of 2.5 (corresponding to 1.0 magnitudes). The resulting histogram is shown in fig. 6, which clearly shows that the shapes of the light curves are incompatible with each other.

As a final test, one event from SS Cygni’s light curve was fitted with a binary lens model. SS Cygni’s light curve was smoothed by averaging over every four points thereby getting estimates of the measurement errors and reducing computation time by a factor of four. Not surprisingly, the “best” fit has a χ^2 of 9,550 with 50 degrees of freedom, which clearly shows that the event in SS Cygni can not be modeled by binary lenses. As can be seen from fig. 7, the modeled light curve not only has more than one peak, but show no resemblance to the observations whatsoever. It is again concluded that cataclysmic variables can not be confused with binary microlens candidates.

6 Conclusion

It has been demonstrated that the developed numerical method reliably finds the global minimum in all the simulated data. The program also determines a best fit model to the binary lens candidate DUO 2 which is compatible with the models published in Alard et al. (1995).

SS Cygni was chosen as an example of a typical cataclysmic variable star, and it was demonstrated that not only is it highly unlikely that the observed events can be explained by binary microlens models, but, due to the difference between the shapes of the observed events and the models, the two phenomena are incompatible with each other when the light curve is well sam-

pled. The attempt to fit one of the observed events with a binary lens model was, as expected, a complete failure.

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